Spacecraft Doppler Tracking as a Xylophone Detector of Gravitational Radiation

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Abstract

We discuss spacecraft Doppler tracking for detecting gravitational waves in whit.1, Doppler data recorded on the ground are linearly combined with Doppler measurements made on board a spacecraft. By using the four-link radio system first proposed by Vessot and Levine [1], we derive a new method for removing from the combined data the frequency fluctuations due to the Earth troposphere, ionosphere, and mechanical vibrations of the antenna on the ground. our method provides also a way for reducing by several orders of magnitude, at selected Fourier components, the frequency fluctuations of the clock on the ground, the clock 011 hoard the spacecraft, the spacecraft antenna, the buffeting of the probe by non gravitational forces, the trail.w[litter on the ground, and the transmitter cm board.

At these frequencies we find a remaining non-zero gravitational wave signal, making this Doppler tracking technique the equivalent of a xylophone detector of gravitational radiation. In the assumption of flying an atomic clock and calibrating the frequency flue.tuaticmw induced by the interplanetary plasma, we estimate for this detector a strain sensitivity equal to $4.7 \times$ l(J^{-19} at 10^{-4} Hz. Experiments of this kind could be performed (at minimal extra cost) with future interplanetary missions by adding radio hardware to the spacecraft payload and to the ground antenna.

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I Introduction

The direct detection of gravitational waves is one of tile most challenging experimental efforts in physics today. A successful observation will not only represent a great triumph in experimental physics, but will also provide a new observational tool for obtaining a better and deeper understanding about their sources, as well a.. a unique test of the various proposed relativist.ic theories of gravity [2].

Since the first experiments by Joseph Weber [3] at the University of Maryland in the early sixties, designs for Earth-based as well as space-bawd detectors have been developed in the form of feasibility st udies, prototypes, or fully operational instruments.

Earth-based detectors, such as resonant bars and laser interferometers, are most sensitive to gravitational waves in the frequency band ranging from about 10 Hz to about 10 kHz, being limited below 10 Hz by seismic noise and shove 10 kHz by instrumental noise [2].

Space-based detectors, such as the coherent microwave tracking of interplanetary spacecraft [4] and proposed Michelson optical interferometers in planetary orbits [5], on the other hand, arc most sensitive to a complementary band of frequencies, effectively in the range from about 10^{-5} Hz to about 1 Hz.

In present single-s])ac.ccraft Doppler tracking observations in particular, 1JIWI% of the noise sources can be either reduced or calibrated by implementing appropriate microwave frequency links and by using specialized electronics. The fundamental instrumental limitation is imposed by the frequency (time-keeping) fluctuations inherent to the reference clocks that control the microwave system [6]. 11 ydrogen maser clocks, currently used in Doppler tracking experiments, achieve their best performance at about 1000 seconds integration time, with a fractional frequency stability of a few parts in 10^{-16} [7]. 'his is the reason why these interferometers in space arc most sensitive to milliHertz gravitational waves. This integration time is also comparable to the propagation time to spacecraft in the outer solar system. The frequency flue.l,uatic)]ls induced by the intervening media have severely limited the sensitivities of these experiments. Among all the propagation noise sources, the troposphere is the largest and the hardest to calibrate to a reasonably low level. Its frequency fluctuations have been estimated to be as large as 10^{-13} at 1000 seconds integration time [8].

In order to systematically remove the effect of the troposphere in the Doppler data, it was pointed out by Vessot and Levine [1] and Smarr *et* al. [9] that by adding to the spacecraft payload a highly stable frequency standard, a Doppler read-out system, and by utilizing a transponder at the ground antenna, one could make

Doppler one-way (Earth-to-spacecraft, spacecraft-to Earth) as well as two-way (spacecraft- Earth-spacecraft, Earth-spacecraft- Earth) measurements. This way of operation makes the Doppler link totally symmetric and allows, by properly combining the Doppler data recorded on the ground with the data measured on the spacecraft, the complete removal of the frequency fluctuations due to the Earth troposphere ionosphere, and mechanical vibrations of the ground antenna from the new formed data. Their proposed scheme relied on the possibility of flying a Hydrogen maser on a dedicated mission. Although current designs of hydrogen masers have demanding requirements in mass and power consumption, it seems very likely that by the beginning of the next century new space-qualified atomic clocks, with frequency stability of a few parts in 10^{-16} at 1000 seconds integration time will be available. They would provide a sensitivity gain of almost a factor of one thousand with respect to the best performance crystal-driven oscillators. Although this clearly would imply a great improvement in the technology of space horn clocks, it would not allow us to reach a strain sensitivity better than a few parts 10^{-16}. This sensitivity would be only a factor of five or ten better than the Doppler sensitivity expected to be achieved on the future Cassini project, a NASA mission to Saturn, which will take advantage of a high radio frequency link (32 GHz) in order to minimize the plasma noise, and will use a purposely built water vapor radiometer for calibrating up to ninety-five percent the frequency fluctuations due to the troposphere [7].

in this paper we adopt the radio link configuration first envisioned by Vessot and Levine [1], but we combine the Doppler *responses* measured on board the spacecraft and on the ground in a different way, as will be shown in the following sections.

In Section 11 we derive the response functions of the two-way Doppler tracking data (Earth-spacecraft-Earth and spacecraft- Earth-spacecraft) measured on the ground and spacecraft. After describing the transfer functions of the noise sources affecting the sensitivities of the two data set, we show that, in a properly chosen linear combination of the two two- way Doppler data, frequency fluctuations induced by the troposphere, ionosphere, and mechanical vibrations of the ground antenna can be removed at all time. We point out that our method is different from the one suggested by Vessot and Levine [1], Smarr *et al.* [9], and Piran *et* al. [10] in that it does not utilize any one-way Doppler measurements (Earth-spacecraft or spacecraft-Earth). Furthermore our technique allows us to reduce by several orders of magnitude, at selected Fourier components, the noise due to the two clocks, to the vibrations of the spacecraft antenna, to buffeting, and to the two radio transmitters. In Section III we estimate the strain sensitivity of this *xylophone* and find it to be equal to 4.7 x 10^{-19} when an atomic clock is used on board. This sensitivity degrades to 2.0 x 10^{-18} when a crystal oscillator is used instead. These figures assume the use of dual frequency links for removing the frequency fluctuations induced by the interplanetary plasma, and an observing time of forty days.

At this level of sensitivity, gravitational wave bursts, continuous signals of known frequency from selected sources, and a stochastic background of gravitational radiation should be detectable. Finally in Section IV we present our comments and conclusions.

II Doppler Tracking as a Xylophone Detector

In the Doppler tracking technique for searching for gravitational radiation a distant interplanetary spacecraft is monitored from Earth through a radio link, and the Earth and the spacecraft act as free falling test particles. A radio signal of nominal frequency ν_0 is transmitted to the spacecraft, and coherently transponded back to Earth where the received signal is compared to a signal referenced to a highly stable clock. Relative frequency changes $\Delta\nu/\nu_0$, as functions of time, are measured. When a gravitational wave crossing the solar system propagates through the radio link, it causes small perturbations in $\Delta\nu/\nu_0$, which are replicated three times in the Doppler data with maximum spacing given by the two-way light propagation time between the Earth and the spacecraft [6]. If we introduce a set of Cartesian orthogonal coordinates (X, Y, Z) in which the wave is propagating along the Z-axis and (X, Y) are two orthogonal axes in the plane of the wave (see Figure 1), then the Doppler response at time t can be written in the following form [6,8,11]

$$\left(\frac{\Delta\nu(t)}{\nu_0}\right)_E \equiv y_E(t) = -\frac{(1-\mu)}{2}\,h(t) - \mu\,h(t-(1+\mu)L) + \frac{(1+\mu)}{2}\,h(t-2L) +$$
$$+ C_E(t-2L) - C_E(t) + 2B(t-L) + T(t-2L) + T(t) +$$
$$+ A_E(t-2L) + A_{sc}(t-L) + TR_{sc}(t-L) + EL_E(t) + P_E(t)\ , \qquad (1)$$

where h(t) is equal to

$$h(t) = h_+(t)\cos(2\phi) + h_\times(t)\sin(2\phi)\ . \qquad (2)$$

Here $h_+(t)$, $h_\times(t)$ are the wave's two amplitudes with respect to the (X, Y) axis, (θ,ϕ) are the polar angles describing the location of the spacecraft with respect to the (X, Y, Z) coordinates, μ is equal to cos θ, and L

is the distance to the spacecraft (units in which the speed of light c = 1).

We have denoted by $C_E(t)$ the random process associated with the frequency fluctuations of the clock on the Earth, $B(t)$ the joint effect of the noise from buffeting of the prohe by non gravitational forces and from the antenna of the spacecraft, $T(t)$ the joint frequency fl actuations due to the troposphere, ionosphere and ground antenna, $A_E(t)$ the noise due to the radio transmitter on the ground, $A_{sc}(t)$ the noise due to the radio transmitter on board, $TR_{sc}(t)$ the noise due to the spacecraft transponder, $EL_E(t)$ the noise from the electronics on the ground, and $P_E(t)$ the fluctuations due to the interplanetary plasma. Since the frequency fluctuations induced by the plasma are inverse] y proportional to the squ arc of the radio frequency, try using high frequency radio signals or by monitoring two different radio frequencies, which are transmitted to the spacecraft and coherently transmitted back to Earth, this noise source can be suppressed to very low levels or entirely removed from the data respectively [12]. Searches for gravitational radiation with Doppler tracking utilizing only one radio frequency are usually performed around solar opposition in order to minimize the frequency fluctuations induced by the plasma [13]. The corresponding time scale during which data are recorded is about 40 days.

From Eq. (1) wc deduce, to first order, that gravitational wave pulses of duration longer than the round trip light time $2L$ give a Doppler response $y_E(t)$ that tends to zero. The tracking system essentially acts as a pass-band device, in which the low-frequency limit f_l is roughly equal to $(2L)^{-1}$ Hz, and the high-frequency limit f_H is set by the thermal noise in the receiver. Since the reference clock an d some electronic components are most stable at integration times around 1000 seconds, Doppler tracking experiments arc performed when the distance to the spacecraft is of the order of a few Astronomical Units. This set the value of f_l for a typical experiment to about 10^{-4} Hz, while the thermal noise gives an f_H of about 3 x 10-2 Hz.

It is important to note the characteristic time signatures of the clock noise $C_E(t)$, of the probe antenna and buffeting noise $B(t)$, of the troposphere, ionosphere, and ground antenna noise $T(t)$, and the transmitters $A_E(t), A_{sc}(t)$. The time signature of the clock noise can be understood try observing that the frequency of the signal received at time t contains clock fluctuations transmitted $2L$ seconds earlier. By subtracting from the received frequency the frequency of the radio signal transmitted at time t, we also subtract clock frequency fluctuations with the net result shown in Eq. (1) [8,9,10].

As far as the fluctuations due to the troposphere, ionosphere, and ground antenna arc concerned, the frequency of the received signal is affected at the moment of reception as well as 21, seconds earlier. Since the frequency of the signal generated at time t does not contain yet any of these fluctuations, we conclude that $T(t)$ is positive-correlated at the round trip light time $2L$ [8,9,10]. The time signature of the noises $B(t), A_E(t), A_{sc}(t)$, and $TR_{sc}(t)$ in Eq. (1) can be understood through similar considerations.

If a clock and Doppler readout system are added to the spacecraft radio instrumentation, and a transponder is installed at the ground station (Figure 2), then Doppler data can be recorded by the spacecraft by coherently tracking the Earth [1]. If we assume the. Earth clock and the on board clock to be synchronized, and we denote with $\bar{\nu}_0$ and $y_{sc}(t)$ the frequency of the radio signal transmitted by the spacecraft and the relative frequency change measured on board the spacecraft at time t respectively, then a gravitational wave pulse appears in the Doppler data $y_{sc}(t)$ with the following signature

$$y_{sc}(t) = -\frac{(1+\mu)}{2}\, h(t) \;+\; \mu\, h(t-(1-\mu)L) + \frac{(1-\mu)}{2}\, h(t-2L) \;+$$
$$+\; C_{sc}(t-2L) - C_{sc}(t) + B(t-2L) + B(t) + 2T(t-L) \;+$$
$$+ A_{sc}(t-2L) + A_E(t-L) + TR_E(t-L) + EL_{sc}(t) + P_{sc}(t) \;, \qquad (3)$$

where the angular dependence of the gravitational wave response in Eq. (3) can be derived from the three-pulse formula (Eq. (1)) by replacing θ with $\pi+\theta$. We have denoted by $C_{sc}(t)$ the random process associated with the frequency fluctuations of the clock on board, $TR_E(t)$ the noise due to the transponder on the ground, $EL_{sc}(t)$ the noise from the electronics on the spacecraft, and $P_{sc}(t)$ the fluctuations due to the interplanetary plasma. The frequency fluctuations due to the transmitters on the ground and on board have been denoted with the same random processes ($A_E(t)$ and $A_{sc}(t)$ respectively) we introduced in Eq. (1). This is correct as long as the two radio signals of frequencies ν_0 and $\bar{\nu}_0$ are amplified within the operational bandwidth (typically forty to fifty megahertz) of the same transmitters [15,16,17]. The Doppler data $y_{sc}(t)$ is then time tagged, and telemetered back to Earth in real time or at a later time during the mission.

let us now introduce a new Doppler data, $y(t)$, defined as the following linear combination of the two data set $y_E(t)$ and $y_{sc}(t)$

$$y(t) \equiv y_E(t) - \frac{1}{2}\left[y_{sc}(t-L) + y_{sc}(t+L)\right] \;. \qquad (4)$$

After some algebra we find that the so formed new Doppler data does not contain any frequency fluctuations due to the troposphere, ionosphere, and antenna mechanical. The possibility of removing noise sources due to the Earth was first derived in a different way by Vessot and Levine [1]. In their scheme they linearly combined t we-way Doppler with "one-way" Doppler, the latter being measurements in which a signal is transmitted from the Earth (the spacecraft) and received by the spacecraft (the Earth). Our method, however, represents a further improvement on their approach since, as we will show below, it allows us to reduce) by several orders of magnitude, at selected Fourier frequencies, the noise due to the clocks, the spacecraft antenna, the buffeting of the probe by non gravitational forces, and the transmitters.

In order to derive this result, let us define $\widetilde{y}(f)$ to be the Fourier transform of the time series $y(t)$

$$\widetilde{y}(f) \equiv \int_{-\infty}^{+\infty} y(t)\ e^{2\pi i f t}\, dt\,. \tag{5}$$

By taking into account Eq. (1) and Eq. (3), we deduce the following expression for the Doppler response $y(t)$ (Eq. (4)) in the Fourier domain

$$\begin{aligned}
\widetilde{y}(f) = {} & i\ e^{2\pi i f L} \sin(2\pi f L)\ \Big\{ 2 \left[\widetilde{C_E}(f) - \widetilde{C_{sc}}(f) \cos(2\pi f L)\right] + \left[\widetilde{A_E}(f) - \widetilde{A_{sc}}(f)\, e^{2\pi i f L}\right] \Big\} + \\
& + 2\ \widetilde{B}(f)\ e^{2\pi i f L}\ \sin^2(2\pi f L) + \left[\widetilde{TR_{sc}}(f) - \widetilde{TR_E}(f)\ \cos(2\pi f L)\right]\ e^{2\pi i f L} + \\
& + \left[\widetilde{EL_E}(f) - \widetilde{EL_{sc}}(f) \cos(2\pi f L)\right] + \left[\widetilde{P_E}(f) - \widetilde{P_{sc}}(f)\ \cos(2\pi f L)\right] + \\
& + \left[R(\mu, f) - R(-\mu, f) \cos(2\pi f L)\right] \widetilde{h}(f)\ ,
\end{aligned} \tag{6}$$

where $R(\mu, f)$ is the transfer function of the three-pulse response [14]

$$R(\mu, f) = -\frac{(1-\mu)}{2} - \mu\ e^{2\pi i (1+\mu) f L} + \frac{(1+\mu)}{2}\ e^{4\pi i f L}\,. \tag{7}$$

Among all the remaining noise sources included in Eq. (6), the frequency fluctuations due to the clock on board the spacecraft are expected to be the largest. The frequency stability of a crystal-driven oscillator, like the onc built for the Cassini mission, for instance, has been measured to be equal to 9.5×10^{-14}, at 1000 seconds integration time, making this clock the most stable space-qualified reference frequency ever built; it is called Ultra Stable Oscillators (USO). Advances in the field of space-based metrology suggest that by the beginning of the next century space-qualified atomic clocks with sensitivities comparable to ground-based clocks will be available [18].

For the moment we will not make any assumptions on the frequency stability of the clock on board, and return to this point later. We will focus instead on the structure of the noise sources shown in Eq. (6). We note that the transfer functions of the combined clock noises, of the combined transmitter noises, and of the noise due to the spacecraft antenna and buffeting, are equal to zero at frequencies that are multiple integers of the inverse of the round trip light time. If we denote by P the time interval over which the Doppler data are recorded (typically forty days), the frequency resolution Af of our data will be equal to $1/P$. This implies that the frequency fluctuations of the clocks, the transmitters, the spacecraft antenna, and the, buffeting of the probe can be minimized at the following frequencies

$$f_k = \frac{k}{2L} \pm \frac{\Delta f}{2} \; ; \qquad k = 1,2,3, \ldots . \tag{8}$$

At these frequencies, and to first order in Af L, the Doppler response $\widetilde{y}(f_k)$ is equal to

$$\begin{aligned} \widetilde{y}(f_k) \approx \mu \left(1 - e^{-\pi i k \mu}\right) \left(1 - (-1)^k e^{\pi i k \mu}\right) \widetilde{h}(f_k) + i\,(-1)^k\, \pi\, \Delta f L \Big\{ 2 \left[\widetilde{C_E}(f_k) - \widetilde{C_{sc}}(f_k)\,(-1)^k\right] + \\ + \left[\widetilde{A_E}(f_k) - \widetilde{A_{sc}}(f_k)\,(-1)^k\right] \Big\} + \left[\widetilde{TR_{sc}}(f_k) - \widetilde{TR_E}(f_k)\,(-1)^k\right] (-1)^k + \\ + \left[\widetilde{EL_E}(f_k) - \widetilde{EL_{sc}}(f_k)\,(-1)^k\right] + \left[\widetilde{P_E}(f_k) - \widetilde{P_{sc}}(f_k)\,(-1)^k\right] . \end{aligned} \tag{9}$$

Note that the fluctuations due to the spacecraft antenna and to buffeting have not been included in Eq. (9) since they are second order in Δf L. If we assume a spacecraft distance L equal to 10 AU, and a tracking time P of 40 days, we find that

$$\frac{\pi\, \Delta f\, L}{c} = 4{,}7 \times 10^{-3}. \tag{1o}$$

In other words, at frequencies which are multiple integers of the inverse of this round trip light time and for a spacecraft out to 10 AU, the noise of the clocks is reduced by about two orders of magnitude, the noise of the transmitters is reduced instead by almost three orders of magnitude, and the noise due to spacecraft antenna and to buffeting is reduced by about five orders of magnitude. This Doppler tracking technique can be regarded as a xylophone detector of gravitational waves, having around one hundred ***"resonant"*** frequencies in the frequency band of a typical Doppler experiment. We should point out, however, that these resonant frequencies in general will not be constant, since the distance to the spacecraft will change over a time interval of forty days. As an example, however, let us assume again $L = 10$ AU, $P = 40$ days, and $f = 10^{-4}$ Hz. The variation in spacecraft distance corresponding to a frequency change equal to the resolution binwidth (3×10^{-7} Hz) is equal to 4.5×10^{6} Km, Trajectory configurations fulfilling a requirement compatible to the one just derived have been observed during paat spacecraft missions [15], and therefore we do not expect this to be a limiting factor. The xylophone technique, namely the idea of looking at selected Fourier components where the transfer functions of some Doppler noise sources are null, has already been implemented successfully by Armstrong [8] in regular spacecraft Doppler searches for gravitational waves. In that case, however, he could reduce by several orders of magnitude at selected Fourier frequencies only the fluctuations duc to the troposphere, ionosphere, and ground antenna ($T(t)$).

Eq. (9) shows some peculiar behavior of the remaining gravitational wave signal in the combined Doppler data. Let us denote by $E(\mu, 2k)$ and $O(\mu, 2k-1)$ the antenna patterns corresponding to multiple even and odd integers respectively of the fundamental frequency of the xylophone. Their analytic expressions are as follows

$$E(\mu, 2k) = 4 \mu \sin^2(\pi k \mu) \tag{11}$$

$$O(\mu, 2k-1) = 2 i \mu \sin[\pi(2k-1)\mu] ; \; k = 1,2,3, \ldots . \tag{12}$$

These antenna patterns arc equal to zero not only when $\mu = \pm 1$, but also at $\mu = 0$, that is to say when the direction of propagation of the gravitational wave is orthogonal to the line of sight of the spacecraft. This is due to the fact that for this particular direction the gravitational wave signal and the noise of the clocks have the same Doppler transfer function, and by removing these noises at frequencies multiple of the round trip light time we also remove gravitational wave signals orthogonal to the line of sight [14]. For sources randomly distributed over the sky, like in the case of a stochastic background of gravitational waves, we find that the antenna patterns given in Eqs. (11), (12) have the following variances

$$\sigma_E^2(2k) \equiv \langle |E(\mu, 2k) - \langle E(\mu, 2k)\rangle|^2 \rangle = 2 - \frac{15}{(2\pi k)^2} , \tag{13}$$

$$\sigma_O^2(2k-1) \equiv \langle |O(\mu,2k-1) - \langle O(\mu,2k-1)\rangle|^2\rangle = \frac{2}{3} - \frac{5}{[\pi\,(2k-1)]^2}\ ; \ k = 1,2,3,\ldots\ , \tag{14}$$

where angle brackets denote averages taken over the random variable θ, which we have assumed to be uniformly distributed over the sphere. Note that the variances $\sigma_E^2(2k), \sigma_O^2(2k-1)$ given in Eqs. (13), (14) are monotonically increasing functions of the integer k, with $\sigma_E^2(2k) \approx 3\,\sigma_O^2(2k-1)$. This characteristic modulation of the intensity of a stochastic background in the xylophone data can be used for improving the likelihood of its detection. We will return to this point in the next section when we will compare our estimated xylophone sensitivities to the predicted amplitudes for a stochastic. background.

Before concluding this section we note that, together with the Doppler data y(t), we can derive another Doppler data set from Eq. (4) by interchanging $y_E(t)$ and $y_{sc}(t)$. Let us define $x(t)$ to be the following quantity

$$x(t) \equiv y_{sc}(t) - \frac{1}{2}\,[y_E(t-L) + y_E(t+L)]\ . \tag{15}$$

The Doppler quantity $x(t)$ does not contain the spacecraft antenna and the buffeting noises (B(t)), and in the Fourier domain has the following form

$$\begin{aligned}
\widetilde{x}(f) &= i\ e^{2\pi i f L}\sin(2\pi f L)\ \{2\left[\widetilde{C_{sc}}(f) - \widetilde{C_E}(f)\cos(2\pi f L)\right] + \left[\widetilde{A_{sc}}(f) - \widetilde{A_E}(f)\,e^{2\pi i f L}\right]\} + \\
&+ 2\ \widetilde{T}(f)\,e^{2\pi i f L}\sin^2(2\pi f L) + \left[\widetilde{TR_E}(f) - \widetilde{TR_{sc}}(f)\cos(2\pi f L)\right]e^{2\pi i f L} + \\
&+ \left[\widetilde{EL_{sc}}(f) - \widetilde{EL_E}(f)\ \cos(2\pi f L)\right] + \left[\widetilde{P_{sc}}(f) - \widetilde{P_E}(f)\ \cos(2\pi f L)\right] + \\
&+ [R(-\mu,f) - R(\mu,f)\ \cos(2\pi f L)]\ \widetilde{h}(f)\ .
\end{aligned} \tag{16}$$

If we add and subtract $\widetilde{x}(f)$ and $\widetilde{y}(f)$ we find the following expressions

$$\begin{aligned}
\frac{\widetilde{x}(f) \pm \widetilde{y}(f)}{2} &= \Big\{[R(-\mu,f) \pm R(\mu,f)]\ \widetilde{h}(f) + 2i\,e^{2\pi i f L}\ \sin(2\pi f L)\left[\widetilde{C_{sc}}(f) \pm \widetilde{C_E}(f)\right] + \\
&+ \left[\widetilde{TR_E}(f) \pm \widetilde{TR_{sc}}(f)\right]e^{2\pi i f L} + \left[\widetilde{EL_{sc}}(f) \pm \widetilde{EL_E}(f)\right] + \left[\widetilde{P_{sc}}(f) \pm \widetilde{P_E}(f)\right]\Big\} \times \\
&\times \left[\frac{1 \mp \cos(2\pi f L)}{2}\right] + i\,e^{2\pi i f L}\ \sin(2\pi f L)\left[\widetilde{A_{sc}}(f) \pm \widetilde{A_E}(f)\right]\left[1 \mp e^{(2\pi i f L)}\right] +
\end{aligned}$$

$$+ 2 \left(\widetilde{T}(f) \pm \widetilde{B}(f)\right) e^{2\pi i f L} \sin^2(2\pi f L) \tag{17}$$

Equation (17) implies that the quantities $\widetilde{x}(f)+\widetilde{y}(f)$ and $\widetilde{x}(f)-\widetilde{y}(f)$ are identically equal to zero at even and odd multiple integers of the inverse of the round trip light time respectively. This is an important property since it can be used in a real experiment to assess the magnitude of systematic errors at the xylophone frequencies in the combined data $y(t)$ and z(t).

III Expected Sensitivities

From Eq. *(9) we* can estimate the expected root-mean-squared (r.m .s.) noise level $\sigma(f_k)$ of the frequency fluctuations in the bins of width Δf, around the frequencies f_k (k = 1,2,3 ,.. ,.). This is given by the following expression

$$\sigma(f_k) = [S_y(f_k)\,\Delta f]^{1/2}, \quad k = 1,2,3, \ldots \tag{18}$$

where $S_y(f_k)$ is the one-sided power spectral density of the noise sources in the Doppler response $y(t)$ at the frequency f_k. In what follows we will assume that the random processes representing these noises are uncorrelated with each other, and their one-sided power spectral densities, provided in Table 1, are aa given in the Riley *et al.* report [7]. This document is a summary of a detailed study, performed jointly by scientists and engineers of NASA's Jet Propulsion Laboratory and the Italian Space Agency (ASI) Alenia Spazio, for assessing the magnitude and spectral characteristic of the noise sources that will determine the Doppler sensitivity of the future gravitational wave experiment on the Cassini mission. Included in Table I is also the spectral density of the noise of a crystal-driven ultra stable oscillator (USO). A description of the radio hardware used for implementing a xylophone detector, schematically represented in Figure 2, is given in the Appendix.

If dual radio frequencies in the uplink and downlink are used, then the frequency fluctuations due to the interplanetary plasma can be entirely removed [12]. We will refer to this configuration as MODE 1. If only one frequency is adopted instead, which we will assume to be Ka-Band (32 GHz), wc will refer to this configuration as MODE II. Ka-Band is planned to be used on most of the forthcoming NASA missions, and will be implemented on the ground antennas of the Deep Space Network (DSN) by the year 1999 for the

Cassini mission.

In Figure 3 we plot the r.m.s. $\sigma(f_k)$ of the noise as a function of the frequencies f_k (k = 1,2,3, ..,.), assuming that an interplanetary spacecraft is out to a distance L = 1 AU. For this configuration the fundamental frequency of the xylophone is equal to 10^{-3} Hz.

The MODE I configuration is represented by two curves, depending on whether an atomic clock (circles) or a USO (squares) is operated on board the spacecraft. Sensitivity curves for the MODE II configuration are also provided, again with an atomic clock on board (up-triangles) or a USO (down-triangles). The best sensitivity is achieved in the MODE I configuration and by using an atomic clock on board. At the Fourier frequency f = 10^{-3} Hz the corresponding r.m.s. noise level is equal to 1.3 $\times 10^{-18}$, and it increases to a value of 5.6 $\times 10^{-18}$ when f = 10^{-2} Hz. If we use a USO instead, at f = 10^{-3} Hz the corresponding r.m.s. noise level is equal to 2.0 x 10^{-18}, while at larger frequencies it becomes identical to the sensitivity curve representing the configuration with an atomic clock on board. This is due to the fact that at higher frequencies the thermal noise dominates over all the other noise sources [8]. As far as the two curves for MODE II are concerned, they coincide almost exactly, since in this case the plasma noise at Ka-Band is the largest among all the remaining noise sources [8] entering in the combined Doppler response.

In Figure 4 we turn to the configuration of a spacecraft out to L = 5 AU, with a corresponding lowest xylophone frequency equal to ‘2.0 $\times 10^{-4}$ Hz. At this frequency, with the plasma noise totally calibrated and by operating an atomic clock on the spacecraft, we find a sensitivity equal to 5,7 $\times 10^{-19}$. At f = 2 $\times 10^{-3}$ Hz the sensitivity degrades to 2.0 x 10^{-18}, and it gets to 1.0 $\times 10^{-17}$ at f ☐. 2 x 10^{-2} Hz.

The sensitivity figures, in the case of operating a USO on board and with plasma calibration (squares), are significantly larger than those for the case in which an atomic clock is used (circles). As the distance to the spacecraft increases, the noise of the clocks at the xylophone frequencies increases proportionally. Consequently the noise of the USO begins to dominate over the noise of the atomic clock at those frequencies. The minimum of the curve is equal to 4.5 x 10^{-18} and at f = 4.0 $\times$ 1 0’’³ Hz, while at f = 2.0 x 10^{-4} Hz and f = 2.0 x 10^{-2} Hz we get an almost equal value of 1.0 $\times 10^{-17}$.

In the MODE 11 configuration the two sensitivity curves (up-triangles, down-triangles) are basically identical, with an optimal sensitivity at f = 1.0 x 10 2 Hz equal to 1.0 $\times$ 10–]7.

Finally, in Figure 5 we analyze the configuration of a spacecraft out to L = 10 AU, with a corresponding lowest xylophone frequency equal to 1.0 $\times 10^{-4}$ Hz. In this case an atomic reference frequency on board gives a significantly better sensitivity than the one achievable with a USO in the low frequency region of the accessible bandwidth. We estimate a strain sensitivity of 4.7 x 10^{-19} at $f = 1 \times 10^{-4}$ Hz, and as we move to the high frequency region the sensitivity decays to a value of 5.9 x 10^{-18} at 1.0×10^{-2} Hz (circles).

As a consequence of the intrinsic narrow-band nature of the xylophone detector, the most likely sources of gravitational waves it will be able to detect will be broad-band bursts, and a stochastic background of gravitational radiation. A complete and accurate review of these sources, and estimates of the corresponding event rates of interest in the frequency band of the xylophone, are given in reference [2]. Here we will briefly summarize some of the possible sources that a xylophone detector could observe. A complete analysis of how to improve further the sensitivity by optimally filtering the data when searching for these signals will be the subject of a forthcoming paper.

A collapse of a star cluster to form a supermassive black bole, in the assumption that the energy emitted in the form of gravitational radiation is 10^{-4} times the rest mass of the system, at 10^{-4} Hz and out to 3 Gpc, would emit a signal of characteristic wave amplitude equal to 3.3 $\times 10^{-17}$. Such a signal would be about 77 times larger than the r.m.s. level we expect to achieve with the xylophone at the same frequency, More optimistic assumptions on the efficiency of the energy radiated [2], in which this could be as much as 10^{-2} times the rest mass of the system, would imply a signal- to-noise ratio of 770 at $f = 10^{-4}$ Hz. Out to 3 Gpc it is conceivable that the event rate could be sufficiently high for having a successful observation during the forty days of tracking, although to our knowledge there are no published estimates on this issue.

The fall of small black boles into supermassive black holes, like what might happen at the end of the merger of two galaxies hosting at their centers a black hole, would give origin to a potentially detectable gravitational radiation. in the scenario in which the larger hole has a mass of 10^{8} solar masses, and the mass of the other hole is only 10^{3} solar masses, at 10^{-4} Hz and out to the Virgo cluster of galaxies, the burst emitted during the merger would be observable by the xylophone with a signal-to-noise ratio of about 5. The Virgo cluster, with its 1000 galaxies could provide a high event rate of such signals, taken into account the indirect evidence of the existence of a supermassive black hole at the center of the galaxy M87 [2].

When searching for a stochastic background of gravitational radiation, the xylophone sensitivity at 10^{-4} Hz would imply an energy density per unit logarithmic frequency and per unit critical energy density [2], Ω, equal to 1.3 $\times 10^{-9}$. Several astrophysical sources of a gravitational wave background, like first-order cosmological phase transitions or cosmic strings, are expected to give an Ω of about 10^{-6}, 10^{-7}. Since the spectral density of any background will be modulated in the xylophone data by the two averaged antenna patterns given in Eqs. (13), (14), we conclude that the xylophone will be able to validate or disprove in a unique way the existence of such sources of background radiation.

The six known white-dwarf binary systems in our galaxy are expected tc, emit an almost monochromatic radiation of amplitude comparable to or smaller than the best r.m.s. noise level the xylophone can achieve [2]. By performing a fine-tuned search for the continuous radiation from such systems during spacecraft cruise, when the inverse of the round trip light time (or its harmonics) coincides with the estimated wave's frequency, would allow an important test of the predictions of general relativity and establish a significant benchmark sensitivity level for future planned space-based interferometers [5].

Something missing in this sentence

Besides known galactic binary systems, we could make an all-sky search at the xylophone frequencies for sinusoidal gravitational waves from binary systems containing black holes.. A binary system in the Virgo cluster, with two black holes of masses equal to 103 solar masses, would radiate a continuous signal of characteristic amplitude equal to 3.2 x 10^{-18} at 10^{-4} Hz. This amplitude would be about 8 times larger than the r.m.s. noise of the xylophone at that frequency.

IV Conclusions

We have discussed a method for significantly increasing the sensitivity of Doppler tracking experiments aimed at the detection of gravitational waves. The main result of our analysis, deduced in Eq. (9), shows that by flying an atomic standard and by adding a Doppler extractor on board the spacecraft and a transponder at the DSN antenna, we can achieve, at selected Fourier components, a strain sensitivity of 4.7 x 10^{-19}. This sensitivity figure is obtained by completely removing the frequency fluctuations due to the interplanetary plasma, at a Fourier frequency equal to 10^{-4} Hz. Our method relies on a properly chosen linear combination of the two-way coherent Doppler data recorded on board with those measured on the ground. It allows us to remove entirely the frequency fluctuations due to the troposphere, ionosphere, and antenna mechanical, and for a spacecraft that is tracked for forty days out to 1 AU it reduces by seven orders of magnitude the frequency fluctuations due to the spacecraft antenna and the buffeting of the probe by non gravitational forces, by almost four orders of magnitude the noise due to the radio transmitters, and most importantly it reduces by three orders of rnagnitudc the noise due to the ground and the on board clocks.

The experimental technique presented in this paper can be extended to a configuration with two spacecraft tracking each other through a microwave or a laser link. Future space-based laser interferometric detectors of gravitational waves [5], for instance, could implement this technique as a backup option, if failure of some of their components would make the normal interferometric operation impossible.

As a final note, a method similar to the one presented in this paper can be used in all the classic tests of the relativistic theory of gravity in which one-way and two-way spacecraft Doppler measurements are used as primary data set. We will analyze the implications of the sensitivity improvements that this technique will provide for direct measurements of the following quantities such as the gravitational red shift, the second-order relativistic Doppler effect predicted by the theory of special relativity, searches for possible anisotropy in the velocity of light, measurements of the Parametrized Post-Newtonian parameters, and measurements of the deflection and time delay by the sun in radio signals. This research is in phase of development, and will be the subject of a forthcoming paper.

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Appendix

In this Appendix wc provide a general description of the radio hardware shown in the block diagram of Figure 2. For a more comprehensive analysis the reader is referred to [7], and references therein.

The ***11-Maser & Frequency Distribution*** represents the overall contribution of the reference clock itself and the cabling system that takes the signal generated by the H-Maser to the antenna. This can be located several kilometers away from the site of the maser, implying that the need of a highly-stable cabling system is required. It has been shown at JPL that optical-fiber cables would not degrade significantly the frequency stability of the signal generated by the maser [7].

The ***Transmitter*** Chain includes all the frequency multipliers that are needed to generate the desired frequency of the transmitted radio signal, starting from the frequency provided by the maser, It also takes into account the radio amplifier, and the extra phase delay changes occurring between the amplifier and the feed cone of the antenna. The noise due to the amplifier is the dominant one, and it has been characterized in references [16], [17].

The ***Transponders*** are responsible for keeping the phase coherence between the incoming and outgoing radio signals on the spacecraft and the Earth. Their performance depends on the accuracy of tracking of the uplink signal by the phase locked loop, and the noise floor and non-linearities of its electronic components [7], [19].

The ***Receiver Chains*** at the ground and on board tbe spacecraft can be modeled as having white phase fluctuations. The contribution to the overall noise budget from the receiver chain on the ground was due until now to the cabling running from the feed of the antenna to the actual receiver. For the ground antenna this long network of cables will be removed with the use of future beam wave guided antennas. These new antennas will be implemented by the year 2000 by the NASA Deep Space Network [20]. The noise figure given for the receiver chain in 'I'able I assumes that a beam wave guided will be used. The frequency fluctuations of the receiver chain on board the spacecraft is estimated to be smaller because of a simple cabling system. We have assumed, however, that the level of noise of the receiver chain on board is equal to the one on the ground.

As the receiver chains, also the ***thermal noise*** of the system has a white phase noise spectrum, and it is due to the finiteness of the signal-to-noise ratio in the Doppler tracking link [8]. Since the frequency is the derivative of the phase, we deduce that the one-sided power spectral density of the relative frequency changes goes as f^2.

A ***Doppler wad-out system*** on board the spacecraft (denoted with the symbol $\otimes$ in Figure 2) has already been built for the Galileo orbiter, which extract Doppler data from a radio signal transmitted from the atmospheric probe in order to measure wind velocities on descent to Jupiter. The space-based radio technology therefore does not need to be developed [21].

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Figure Captions

Figure 1,

A radio signal of nominal frequency ν_0 is transmitted to a spacecraft and coherently transponder back to Earth. The gravitational wave train propagates along the Z direction, and the cosine of the angle between its direction of propagation and the radio beam is denoted by μ. An equivalent link from the spacecraft to the Earth and back is established, and is symbolically represented with dashed arrows. See text for a complete description.

Figure 2.

Block diagram of the radio hardware at the ground antenna of the NASA Deep Space Network (DSN) and on board the spacecraft (S/C), that allows the acquisition and recording of the two Doppler data $y_E(t)$ and $y_{sc}(t)$. A description of each individual' block in this diagram is provided in the Appendix.

Table I

List of the noise sources entering into the combined Doppler response y(t). The Allan deviation at a given integration time τ is a statistical parameter for describing frequency stability. It represents the root-mean-squared expectation value of the random process associated with the fractional frequency changes, between time-contiguous frequency measurements, each made over time intervals of duration τ. The numbers provided in this table are taken from the *Riley et al.* report [7].

Figure 3.

The r.m.s. noise level as a function of the frequencies $f_k (k = 1, 2, 3, ...)$, estimated for a spacecraft that is out to a distance L = 1 AU. Sensitivity figures for the four distinct configurations are represented with four different symbols. Circles represent r.m.s. values as functions of the xylophone frequency when plasma frequency fluctuations are totally removed and an atomic standard is on board. Squares are sensitivity values again with plasma calibration, but now with a USO on the spacecraft. Up-triangles are used for representing sensitivity figures affected by plasma noise at Ka-Band, and an atomic clock is used on board. Down-triangles assume a plasma noise at Ka-Band, but now with a USO on board.

Figure 4.

As in Figure 3, but now with a distance to the spacecraft $L = 5$ AU. Sce text for a complete description of the sensitivity curves.

Figure 5.

Asin Figure 3, but with L= 10 AU. See text for a complete description of the sensitivity curves.

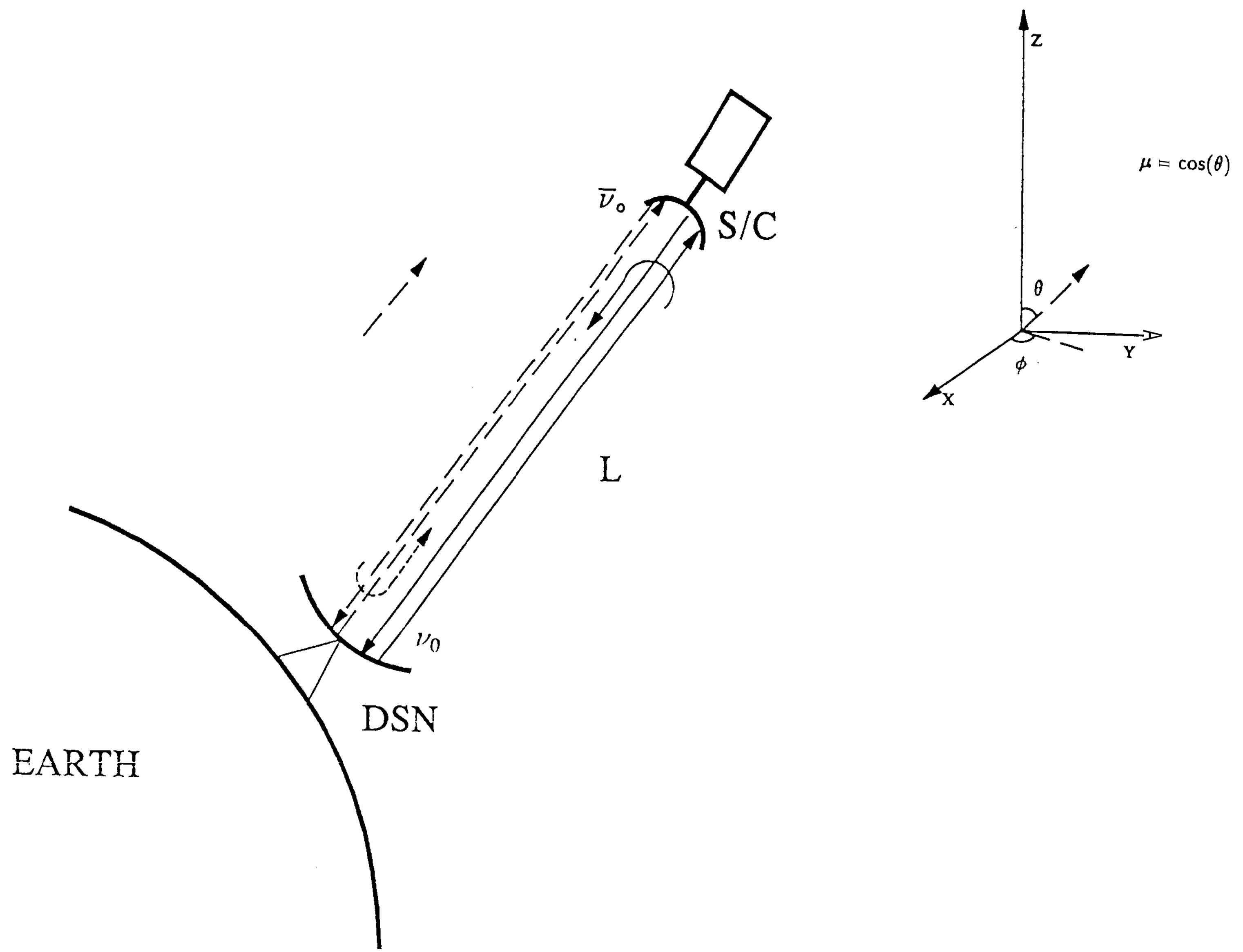

Figure 1.

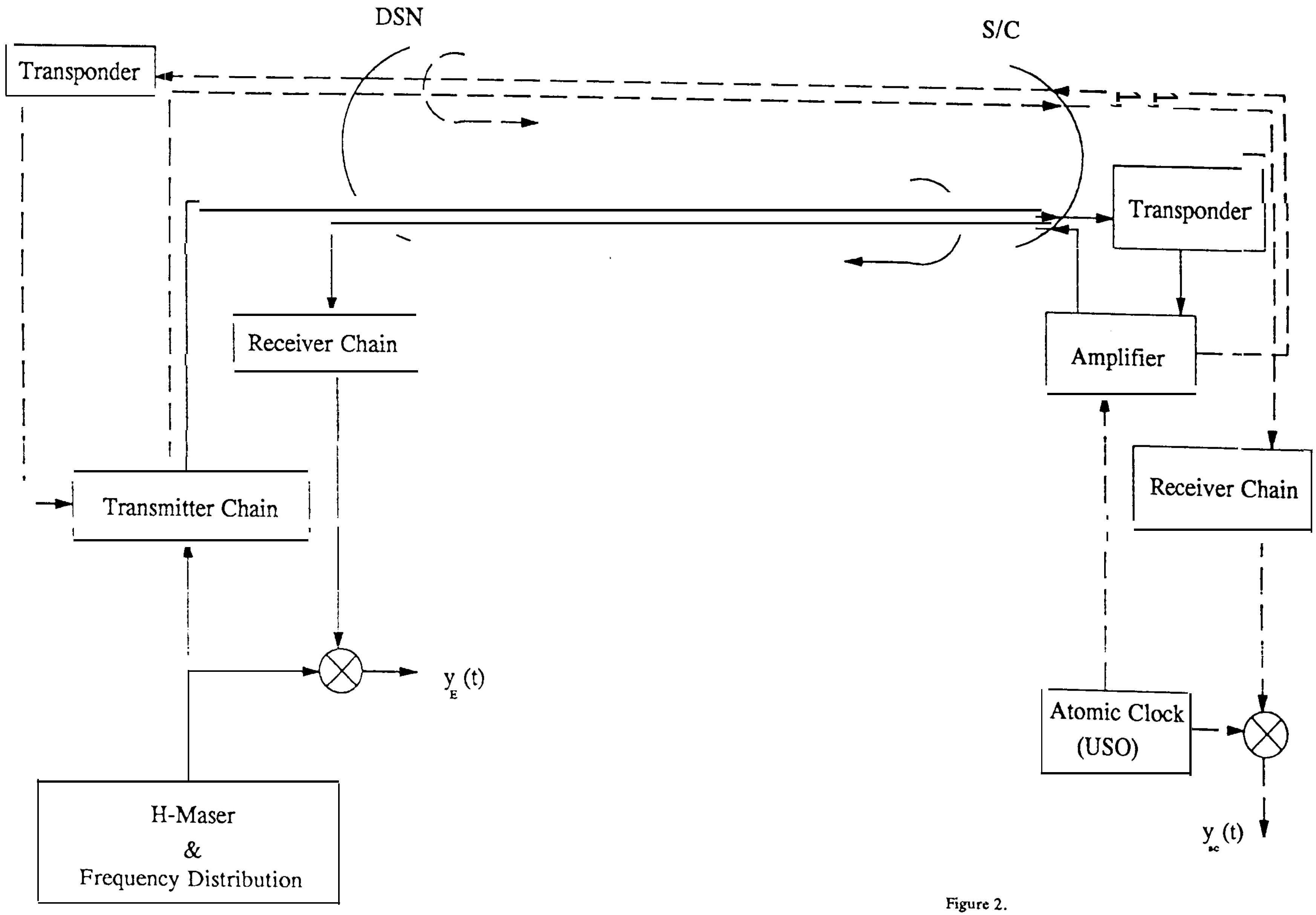
DSN
S/C
Transponder
Transponder
Receiver Chain
Amplifier
Transmitter Chain
Receiver Chain
$y_E(t)$
Atomic Clock
(USO)
H-Maser
&
Frequency Distribution
$y_{sc}(t)$

Figure 2.

Table I

Error Source	Allan Deviation @ 1,000 sec.	Fractional Frequency One-Sided Power Spectral Density
Plasma (@ IQ-Band)	7.0×10^{-16}	5.4×10^{-30} f - m
H-Maser	4.0×10^{-16}	$10^{-26} f + 10^{-31} f^{-1} + 10^{-28}$
Frequency Distribution	1.0×10^{-17}	$1.3 \times 10^{-26} f^{2}$
Receiver chain	3.1×10^{-17}	$1.3 \times 10^{-25} f^{2}$
Trasmitter chain	3.4×10^{-16}	2.3×10^{-28}
Thermal Noise	3.8×10^{-17}	$1.9 \times 10^{-25} f^{2}$
Transponder	4.0×10^{-17}	$2.6 \times 10^{-27} f$
Amplifier	5.0×10^{-17}	$4.0 \times 10^{-27} f$
USO	9.5×10^{-14}	$6.5 \times 10^{-27} f^{-1}$

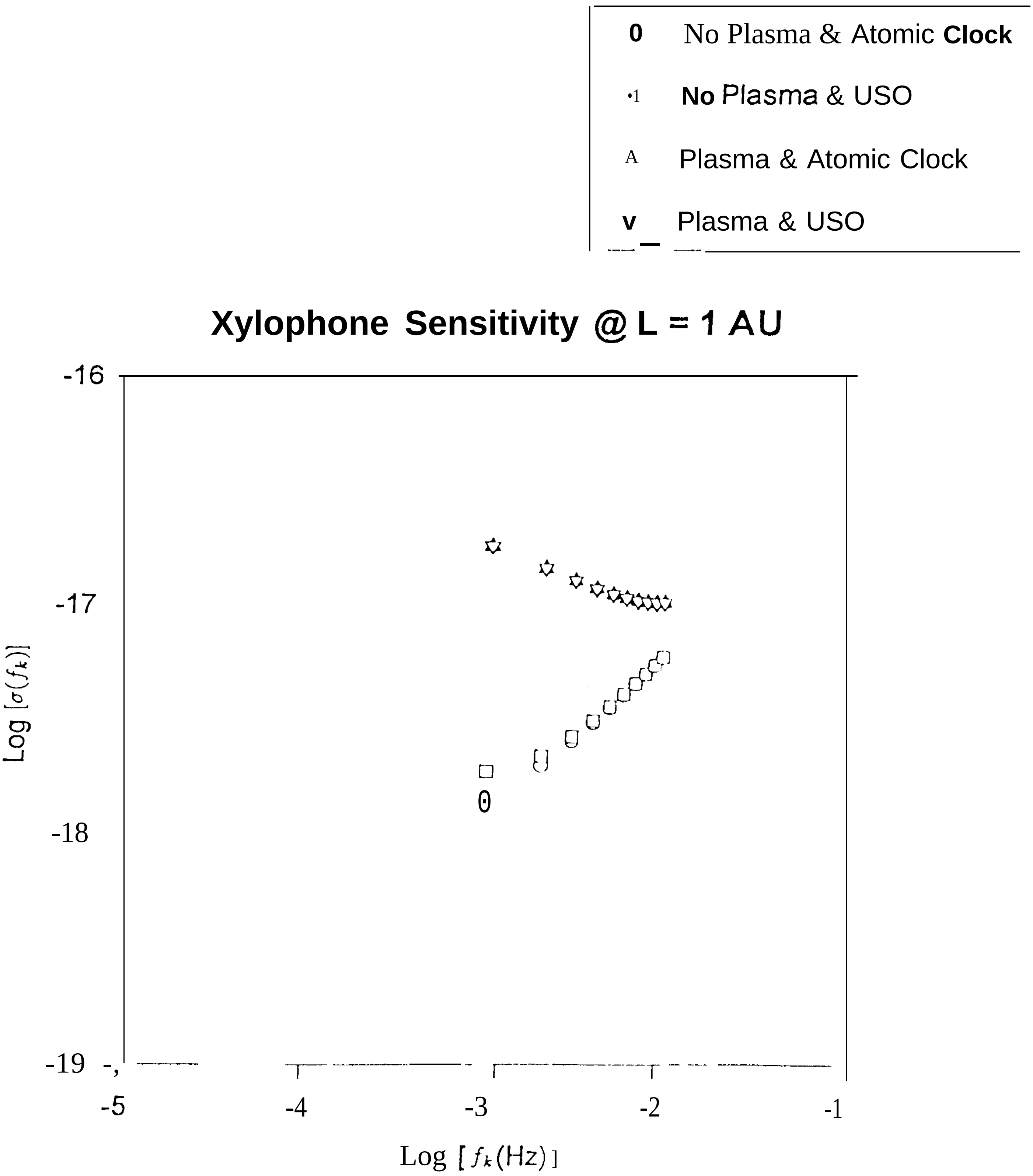
0 No Plasma & Atomic Clock
•1 No Plasma & USO
A Plasma & Atomic Clock
v Plasma & USO
Xylophone Sensitivity @ L = 1 AU
-16
-17
-18
-19
-5
-4
-3
-2
-1
Log [σ(f_k)]
Log [f_k(Hz)]

Figure 3.

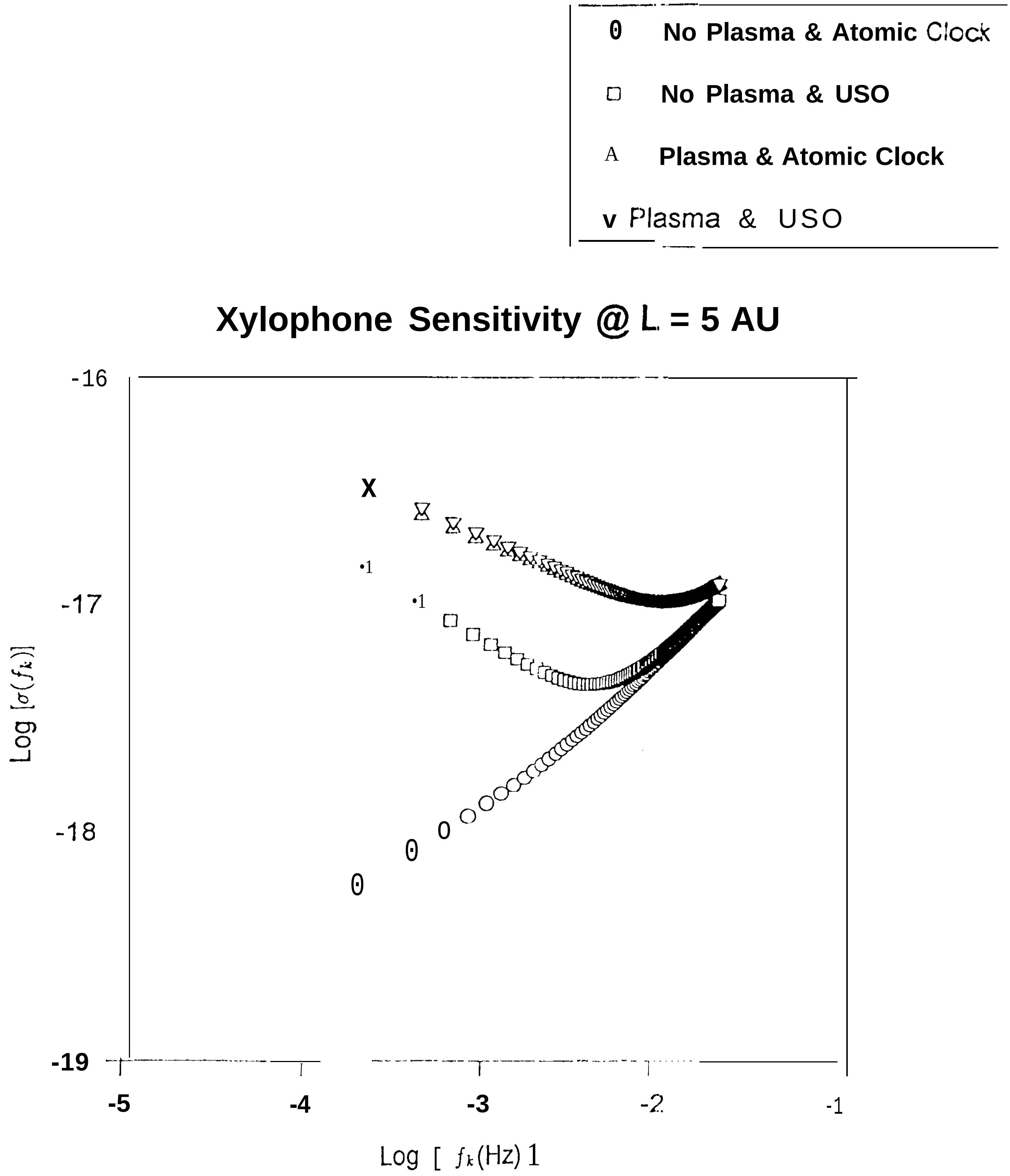
0 No Plasma & Atomic Clock
No Plasma & USO
A Plasma & Atomic Clock
v Plasma & USO
Xylophone Sensitivity @ L = 5 AU
-16
-17
-18
-19
Log [σ(fk)]
-5
-4
-3
-2
-1
Log [fk(Hz) 1

Figure 4.

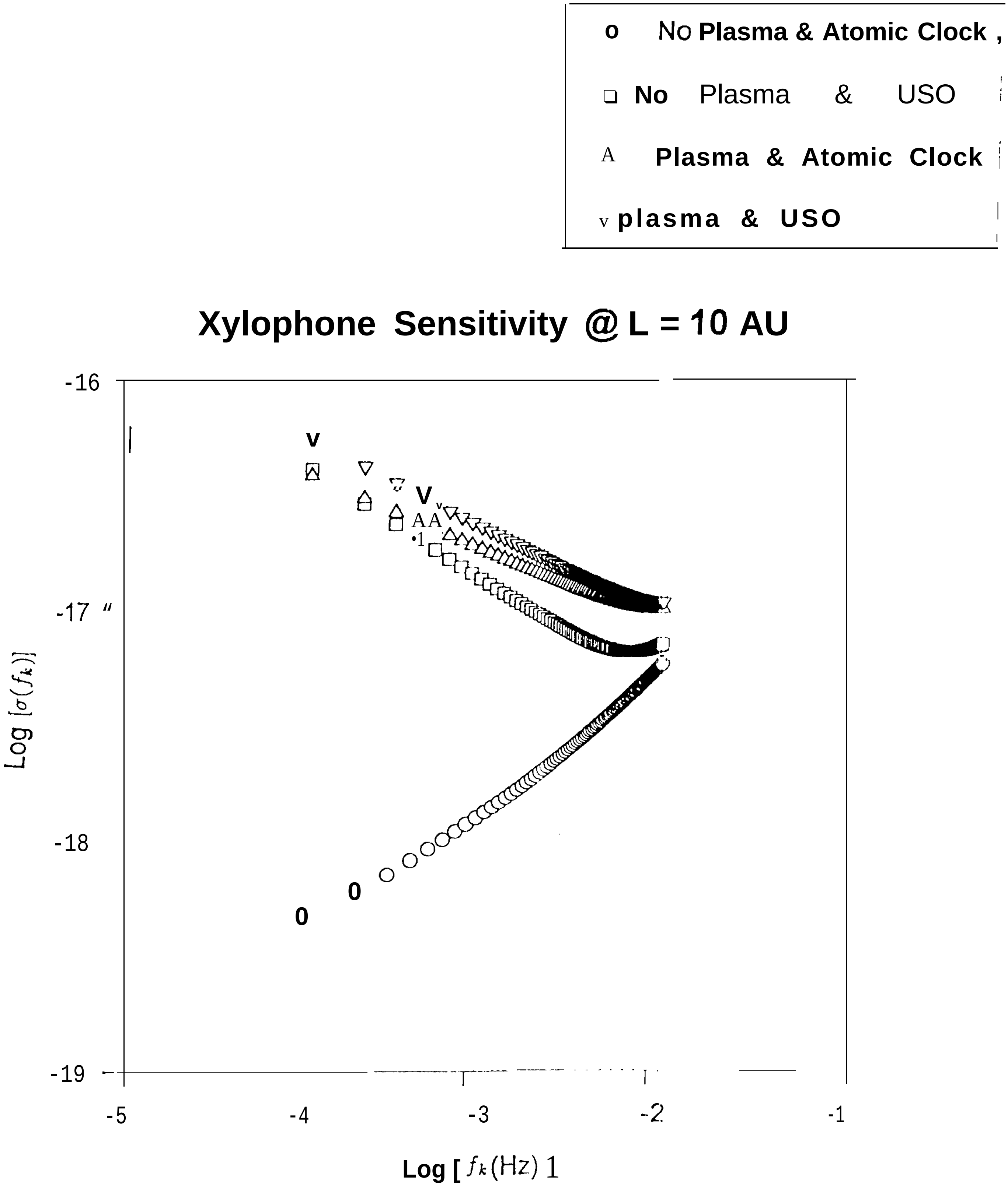
No Plasma & Atomic Clock
No Plasma & USO
Plasma & Atomic Clock
plasma & USO
Xylophone Sensitivity @ L = 10 AU
-16
-17
-18
-19
Log [σ(fk)]
-5
-4
-3
-2
-1
Log [fk (Hz)]

Figure 5.

CLEARANCE NO. PHW108-9/19/95

Spacecraft Doppler Tracking as a Xylophone Detector of Gravitational Radiation
By:- Massimo Tinto

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COMMENTS: The innovator claims a new technique for improving Doppler Tracking, however, the application appears limited to detection of gravitational waves, hence, patent action is not recommended.

PREPARED BY: P.H. WARE September 19, 1995

(08/09/91)